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Michael Titelbaum, *Quitting Certainties: A Bayesian Framework Modeling Degrees of Belief*. Oxford: Oxford University Press, 2013. xii + 345 pp.

Many philosophers have suggested that a fruitful way to think about belief and uncertainty is through the framework of probability theory. The basic model for this way of thinking, often called “Bayesian,” is that an agent’s degrees of belief can be represented by a probability function, which changes by incorporating new information as certain. The title of this book is a reference to the fact that it includes a new Bayesian framework that can deal with the loss of certainties, which can often happen either because of forgetting, or because of context sensitivity. (For instance, I am now certain that it is Friday, but in a few hours I won’t be, because it will be false in the new context.)

However, while this new framework is important, and makes this book essential reading for people interested in the epistemology of these situations (whether Bayesian or not), the first seven chapters make an important contribution of their own—one that deserves to be more broadly noted. In these chapters, Titelbaum spells out one of the most systematic formulations of how Bayesian epistemology should be applied. Many of the vexing problems for it (or indeed for any attempt to use formal methods in any area of philosophy) are made much less vexing by means of a careful setting up of the formal machinery.

Titelbaum is careful to distinguish the actual requirements of rationality from the formal statements of his system, and chapter 2 provides some extensive discussion of the possible relations between the two. Chapter 3 explicitly presents his formal system as a kind of *model* of the requirements of rationality—it is not intended to represent precisely what these requirements actually are in all cases but is instead supposed to give us a good idea of what the requirements are like within a sort of “domain of applicability” of the model.

This way of thinking about a formal system is familiar from science: we treat Newtonian mechanics as a good system for reasoning about the behavior of certain types of objects, but we know that it will be bad in the presence of the very large or small, extremely high velocities, and friction or other nongravitational forces. Bayesianism originally arose in the philosophy of science, where this way of thinking may have been more common, but epistemologists may find this idea of a model as approximation less familiar. I think Titelbaum provides a valuable resource in showing that this is a way that philosophical inquiry can proceed as well.

While this review can’t do full justice to Titelbaum’s formalism, I will give a quick overview in order to help illustrate some of the insights it provides and the way in which it should be extended. The basic idea is that when dealing with a “story” (an informal description of a scenario in which some agent’s doxastic state evolves over time), Titelbaum generates a “model,” which includes a language (giving the set of claims for which the agent’s degrees of belief are

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of interest), and a set of times at which we care about the agent's degrees of belief. Titelbaum then provides several "systematic constraints" and suggests that we extract "extrasystematic constraints" from the story and use these constraints together to generate a function P assigning a value $P_i(x)$ for each time i in the time set and sentence x in the language. These assignments are called "verdicts" of the model, and they are interpreted by saying that an agent whose degrees of belief conflict with the verdicts of the model must be in violation of some of the requirements of ideal rationality.

The distinction between systematic and extrasystematic constraints is key to Titelbaum's application of these models. He imposes just four systematic constraints: Subjective Finite Additivity, the Ratio Formula, Generalized Conditionalization, and the Proper Expansion Principle. (These are helpfully summarized in a quick reference page at the beginning of the book. I will discuss them as they come up.) However, he also requires that for each i and x , there is either an extrasystematic constraint of the form $P_i(x) = 1$ or one of the form $P_i(x) < 1$, depending on whether our understanding of the story indicates that the agent should be certain of x at that time or not. There may also be further extrasystematic constraints if the story is one in which we think further rational requirements may apply (for instance, symmetries of dice rolls or knowledge of chances). Perhaps future developments of this framework will attempt to account for more of these constraints in a systematic manner, but for now this is an honest way of dealing with the limitations of any formal framework.

The first two systematic constraints are synchronic while the other two constraints provide the diachronic rules that apply to updates over time. Finite additivity, together with the extrasystematic constraints requiring probabilities to either equal 1 or be less than 1, is equivalent to probabilism. His version of the ratio formula says that if $P_i(\sim y) < 1$, then the conditional probability $P_i(x|y) = P_i(x \& y)/P_i(y)$, and it is undefined otherwise. As Titelbaum notes, it makes sense for conditional probabilities to be undefined when one is certain that the antecedent is false, which means that this framework is well designed for stories in which rational agents have degree of belief 1 only in things of which they are certain.

However, if we would like to include stories where there are infinitely many alternatives (for instance, for scientific reasoning about numerical parameters), it would be useful to distinguish certainty from degree of belief 1. To describe his generalized conditionalization rule (which I will say more about later), Titelbaum introduces the notion of a "certainty set" at a time i , which he defines as the set of all the x such that $P_i(x) = 1$. However, it seems to me that a slightly more general form of the framework could be defined by taking certainty as a primitive, and adding three more systematic constraints—one requiring certainties to be closed under logical deduction, another saying that if an agent is certain of x at time i , then $P_i(x) = 1$, and another saying that for all i and x , $P_i(x) \geq 0$. It is more natural to treat

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certainty, rather than probability 1, as the feature that must come from extrasystematic constraints, even if this requires further systematic constraints. We also need to modify the ratio formula constraint so that it says that if $\sim y$ is certain at i , then $P_i(x|y)$ is undefined, and otherwise (following Popper) it satisfies $P_i(x \& y) = P_i(x|y)P_i(y)$. I have worked through the examples in Titelbaum's book, and it looks like every case works out just as well under this generalized framework, except for a few where a verdict of " $<$ " is replaced by one of " \leq ."

Of course, the diachronic principles yield most of the interesting insights of the framework. The principle of generalized conditionalization says that if j and k are times whose certainty sets C_j and C_k are compatible, then $P_j(x|C_k) = P_k(x|C_j)$. Among other things, this generates the traditional requirement of update by conditionalization in the case where an agent learns new certainties at a later time without losing any others. Furthermore, as Titelbaum shows in chapter 6, this generalized principle can derive plausible verdicts in various cases of memory loss. This chapter also tries to explain the failure of the "reflection principle" in these cases, though I think this discussion is not fully successful (particularly because reflection involves second-order beliefs about one's own beliefs, which his framework can't accommodate).

At this point, before Titelbaum discusses the fourth systematic rule (which is the one essential for treating many other cases of the sort suggested by the title) comes chapter 7, which in my view poses some of the most interesting and important questions in the book. This chapter discusses the notion of diachronic consistency that is implicit in generalized conditionalization and considers what justification it could have. As Titelbaum shows, there are several even stronger principles that entail it. For instance, if there is one objective rational ur-prior probability distribution such that at any time, every rational agent ought to have degrees of belief that equal the values of this distribution conditionalized on their certainties, then rational agents will satisfy generalized conditionalization. In fact, generalized conditionalization would even hold when the degrees of belief at different times belong to different rational agents! And in fact, many different views on which the evidence one has at a time uniquely determines the rational degrees of belief to have at that time will justify generalized conditionalization.

The more difficult question is how to justify this norm of diachronic consistency within a more permissive framework, on which one's evidence does *not* uniquely determine the degrees of belief one should have. It may be that there is no requirement that one *actually* update in ways that satisfy this rule but merely that one must be *committed* to updating in such a way. Or perhaps, that updating in this way is part of what it takes to count as the *same* rational agent over time. The question is not answered here, but I think this chapter raises some very important considerations that all theories of diachronic rationality must confront. Some of Titelbaum's discussion here relies heavily on the work of

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Isaac Levi, particularly from his book *The Enterprise of Knowledge* (1980). This book has been underappreciated by philosophers working in the tradition of Richard Jeffrey and David Lewis, and hopefully Titelbaum's work will help draw more attention to the insights contained within it and help make them more accessible to new audiences.

The remainder of the book discusses the many issues that arise when context sensitivity occurs in updating—either alone, or in combination with actual or possible memory loss. Learning first that it is Friday and later that it is Saturday is not like an update involving new information. One is not learning the unexpected thing that it is actually Saturday at the time that one once thought was Friday but is in fact just learning that the eventually expected change in context has now occurred.

The specific case of this sort that is most heavily discussed in recent epistemology is that of Sleeping Beauty. In this case, Beauty is a participant in a science fiction psychological experiment, and she knows in advance all the details of how it will be structured. On Sunday night, the experimenters will flip a fair coin. On Monday, she will wake up and be debriefed by the experimenters. On Monday night, when she is asleep, she will either be given drugs that cause her to dreamlessly sleep all through the day on Tuesday to wake up on Wednesday (if the coin came up heads), or be given different drugs that erase her memories of Monday, so that she wakes up on Tuesday in the same mental state that she woke up in on Monday, and she will be debriefed by the experimenters again. When she wakes up during the experiment, given what she has been told, she does not know whether it is Monday or Tuesday, and she does not know whether the coin came up heads or tails.

Some philosophers have argued that she has learned nothing relevant to the coin toss, so she should have degree of belief $\frac{1}{2}$ that the coin came up heads. Others have argued that since she wakes up twice as many times if the coin comes up tails, she should have degree of belief $\frac{1}{3}$ that the coin came up heads. Titelbaum gives one of the first fully formal systems that allows one to calculate the answer ($\frac{1}{3}$), rather than reasoning intuitively. (As he points out, there are other formal systems now that give competing answers.)

Titelbaum's answer comes from his "proper expansion principle." To explain this principle, we have to define the notion of an "epistemically context-insensitive sentence." This is a sentence in the language of the model such that at every time in the model, the relevant agent is certain that the sentence has the same truth value at all times in the model. Such a sentence may in fact be context-sensitive, as long as the context sensitivity is not relevant within the particular story being modeled. For instance, we might consider a story that discusses Beauty's degrees of belief on Monday, from the morning when she first wakes up, until the afternoon when she has been told that it is Monday (but not how the coin came up). In this case, the sentence "today is Monday" is epistemically context-insensitive, even though

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it does use the context-sensitive expression “today.” It would be epistemically context-sensitive in a story that includes either Sunday or Tuesday as well, so this notion is relative to the story.

The proper expansion principle says that if every epistemically context-sensitive sentence has insensitive replacements (meaning that at each time in the story, there is some epistemically context-insensitive sentence such that at that time, the agent is certain that the two sentences have the same truth-value), then any verdict calculated in a model using just the epistemically context-insensitive sentences will also be a verdict of the full model. Issues around finding these replacements and stating exactly how this principle applies to a variety of cases are too tricky to go into in great detail here, but Titelbaum discusses them at length in the second half of the book.

One interesting feature of this book is that, unlike many other philosophical works, it doesn’t try to derive its results from first principles. Instead, the major motivation Titelbaum gives for his framework is just that it is a systematic account that appears to give the right answer in many cases. Importantly, he doesn’t just focus on the problem cases that have been central to the literature but discusses many simple cases, like people losing track of time while their alarm clocks fail. It is only after showing that he can properly account for these cases that he then applies the framework to *Sleeping Beauty*. This is related to the central idea that what he is doing here is “modeling” situations within a certain domain of applicability. This may not be satisfying to philosophers who are interested in giving full explanations, but this may be a necessary trade-off for creating a workable formal system.

Still, given the way that the Proper Expansion Principle has been set up, there are some beginnings of a justification. In response to a challenge from Sarah Moss (discussed toward the end of chapter 8), Titelbaum has modified this principle from how it worked in his previous papers on the topic. Now the claim is not that one can derive verdicts in *any* sublanguage where all removed sentences have equivalents but rather only if the sublanguage consists of the epistemically context-insensitive sentences. This suggests a motivation. One might be able to argue that the epistemically context-insensitive claims are the ones that drive our updates—these are the ones that reflect our information about the world. But even without a full justification, chapter 10 of this book provides a useful summary and comparison of this system with those of Joseph Halpern, Chris Meacham, Sarah Moss, and Robert Stalnaker.

Given the modification I have suggested above to the way the system treats certainty, I think it would also be interesting to see how well it can deal with various infinitary cases. For instance, Jacob Ross’s “St. Petersburg *Sleeping Beauty*” case would be interesting to evaluate, to see if the reasoning he suggests to lead toward violations of countable additivity does in fact go through on this particular system.

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In closing, this book provides an interesting and systematic account for dealing with the gain and loss of certainties and explaining how they can rationally drive all the changes in degree of belief that an agent goes through over time. Even if the system doesn't turn out to be the right way to think about updating of degrees of belief in light of memory loss and context sensitivity, I think Titelbaum's care in setting up the system makes it an admirable model for any future development that aims to replace it, either for these purposes or even for the more restricted purposes to which Bayesianism is usually applied.

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Kok-Chor Tan, *Justice, Institutions, and Luck*.

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Luck egalitarianism—the thought that unchosen forms of advantage are presumptively unjust—has not been the most fashionable view as of late. The rise of the “democratic equality” of Elizabeth Anderson—along with the continuing influence of John Rawls's own political view of justice—have contributed to the sense that luck egalitarianism is simply too blunt an instrument with which to analyze social and political justice. Kok-Chor Tan attempts, in this recent book, to develop a chastened and moderate luck egalitarianism, one that might demonstrate why luck egalitarianism is attractive—and how it can defend itself against its critics.

The book is a model of clarity and rigor; it is a thin volume, but it manages to cover several decades of recent thinking about social justice, and do so with grace and accuracy. It would be of use, even if Tan's own arguments were rejected, as a useful supplement to any class on recent political philosophy. Tan's own arguments, though, are the real heart of the book, and these arguments amply repay close reading.

Tan develops his view with reference to three questions: *where* does distributive justice matter? *Why* should we care about economic inequality? And among *whom* does justice hold? The first question is taken up in the first third of Tan's book; his answer is that distributive justice applies in the first instance to *institutions*—to those large-scale structures that transform natural facts into inequalities of wealth and income. This part of the book argues that, contrary to the arguments of G. A. Cohen, an institutional focus is not an abdication of moral concern but a recognition of the possibility of value pluralism. Were